Study on Hexagonal Grid in Image Processing

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ABSTRACT

Image processing is very important in several applications and have been using in them very efficiently. Normally we use a rectangular grid for the processing of images. There could be some other approaches to use as an alternate for this. One new approach is to change the grid from rectangular to hexagonal, because of its various advantages over the later. Out of the many advantages for the hexagonal structure in image processing, the primary one is its resemblance with the arrangement of photoreceptors in the human eyes. Due to the change in arrangement the amount of pixels required is very less. There is no inconsistency in pixel connectivity and thus angular resolution is higher in this arrangement. Hexagonal grid will be advantageous in many real time applications. Implementation of Hexagonal grid can be done in various methods including the spiral addressing scheme. Applying hexagonal grid in image processing is very advantageous and easy for mimicking human visual system.

Image processing, Hexagonal grid, Hexagonal addressing scheme, spiral addressing scheme

1   INTRODUCTION

Researchers have been studying the feasibility of introducing the hexagonal grid in the area of image processing. There are various pixels tessellation schemes available to represent digital images. Hexagonal grid is also a pixel tessellation scheme which is efficient than any other schemes. Normally digital images are mapped on square lattice and here we are changing the square lattice to hexagonal lattice for the hexagonal image processing. Hexagonal coordinate system is well suited for creating the artificial human visual system, because the arrangements of the photo receptors in the human retina are in hexagonal form.

The first section in this explains about the reasons for rectangular to hexagonal grid change. The following section describes the various features of hexagonal sampling scheme. In the next section various methods for hexagonal image representation are mentioned. Later section explains about the various addressing schemes and then the conclusion of the present study.

2 RECTANGULAR TO HEXAGONAL GRIDS

In hexagonal lattice number of pixels required to sample a data is very less [1]. Due to the resemblance of human vision with the hexagonal arrangement the computational operations will become faster. And thus it will give a new face in the processing speed and efficiency of image processing operations. There are various works available in the literature which touches this field. All of them points to the advantages of the hexagonal grid with the normal square grid.

Even though there are many advantages, hexagonal grid is not widely used in image processing. Lack of capturing and display devices in hexagonal grid makes it impossible to attain the benefits of hexagonal grid. Due to this limitation researchers are trying to mimic the hexagonal grid on rectangular grid itself. But all of the methods are simulations only and thus we cannot achieve the real advantages of the hexagonal grid. In the real hexagonal structure pixels are not arranged in rows and columns. Researchers introduced various addressing schemes and coordinate systems to implement the hexagonal grid atleast theoretically. Sampling lattice is one aspect of the sensing methodology used in computer vision. Thus we can represent a hexagonal grid of pixels on the existing rectangular screens for modeling and processing purpose, which is more suitable for computer vision modeling. Once sampling lattice is digitized into hexagons, various image processing operations can be performed on these sub-sampled images.

According to Mersereau[2], in a hexagonally sampled image there will be 13.4% fewer sampling points compared with the square grid. All image processing operations can be done on this grid with utilizing all the benefits. Many resampling techniques were proposed like brick wall, quincunx sampling, least squares approximation of splines, etc [3]. One of the resampling techniques to obtain hexagonal grid is to suppress alternate rows and columns from the square

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sampled image[4] as shown in the figure, Fig 1 and the image processing operations can be performed on the resampled hexagonal image.

(0,0)	(0,19)	(0,0)	(0,19)
(9,0)	(19,19)	(9,0)	(9,19)

Fig 1   (a) Rectangular sampling   (b) Hexagonal sub sampling
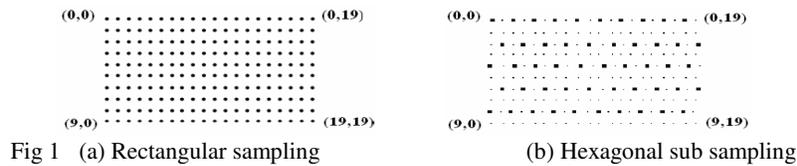
Another resampling method was proposed by Staunton [5] which is to shift the alternate rows of pixels in the image by a half pixel distance.

3 FEATURES OF HEXAGONAL SAMPLING SCHEME

Digitization is one of the hardest tasks which scientists were worried about in image processing. Because the real scene will be in a continuous plane and the images are on a digital screen with discrete points. In the process of digitization, the discrete points referred as pixels have to be arranged on the screen which should be properly addressed. The arrangement of the pixels should be regular and its representation on the plane should be efficient also.

3.1 Regular Tessellation schemes
There are only three tessellations available to tile a plane which is regular and the samples do not overlap among each other and with its gaps. All other tessellation schemes will either be inconsistent in the neighborhood connectivity or will become gaps or overlaps among the samples. The square tessellation is the commonly used one and it uses the Cartesian coordinate system for all operations and thus it is simple. The triangular one gives a tight arrangement than the square one which explains that more information is contained in the same image. The hexagonal one is the most compact and tight packing among the other two. Normally we used to take beehives as the typical example for the tight packing which is in hexagonal arrangement.

3.2 More Efficient Sampling Schemes
Aliasing will occur to the images whose sampling rate is not sufficient. Peterson and Middleton [6] found out the fact that the least samples are required for the reconstruction of a wave number limited signal in hexagonal lattice. From this it is clear that square lattice is not the best one. Mersereau [2] also concluded that signals in Fourier space requires only 13.4% lesser samples to represent the same image data in hexagonal grid compared to the rectangular one. By using this advantage it is clear that the storage space required will become less and the computation cost will also become less. Vitulli [7] also investigated the sampling efficiency using hexagonal grid and concluded that it's the same as Mersereau explained in his work. Vitulli also found out that using the hexagonal grid, wider spectra of signal can be sampled without aliasing with fewer amounts of samples.

3.3 Smaller Quantization Error
Quantization is compulsory for the image processing operations because of the limited capable sensors to represent the real world scenes. Quantization error is an important measure to analyze the merits of the configurations of different types of sensors. Kamgar-Parsi [8,9,10] showed that hexagonal sampling gives lesser quantization error when compared to square.

3.4 Consistent Connectivity
In an image, pixels are arranged adjacent to each other. Every pixel is connected to each other based on some specific criteria [11]. In a square grid, there are two types of pixel neighborhood. They are four-neighborhood and the other one is the eight-neighborhood. So there will be four-way connection or eight-way connection depending upon the number of common edges share among the pixels[12]. In the hexagonal grid there is only six way connectivity. In this there will be only one common edge and two common corners. This will be useful when we are doing the operations such as skeletonisation[12,13]. The connectivity in hexagonal domain is consistent and it is fixed to six way connectivity [14, 15].

3.5 Equidistance
In the normal grid that is the square grid, we have two types of distant measures. The distance between the diagonal pixels are √2 times than that between the horizontal pixels as in Fig 2(a).

<div align="center">(a)   (b)</div>
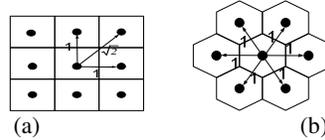
Fig 2 Distance in square grid and hexagonal gird

In hexagonal grid, there is only one consistent connectivity and each pixel has six neighboring pixels. And all the pixels are single unit distant from each other as in Fig 2 (b).

3.6. Greater Angular Resolution

For representing curved images hexagonal grid is efficient. Adjacent pixels in hexagonal grid are separated by sixty degree instead of ninety degree in the existing one. So, curved images can be represented in a better way. Main reason behind this is the consistent connectivity that we have already studied. Human eyes have a special visual preference of seeing the lines which are at oblique angle [16]. So this is another reason for representing lines also in a better way in hexagonal grid.

3.7 Higher Symmetry

Many morphological operations are developed by Serra [17] and are been widely used in Image processing .He studied the same on different grids and identified the fact that hexagonal grid has higher symmetry and simple operations.

3.8 Other Features of Hexagonal Grid

Hexagonal grid closely resembles with the human visual system. The rods and cones are arranged in a hexagonal fashion in the fovea. So in order to mimic any of the human visual system activity we need to be in the hexagonal grid to get the better results.

4   HEXAGONAL IMAGE REPRESENTATION

Even after getting all the advantages of the hexagonal grid it has not been used widely in image processing. The main reason is that there is no image capturing and display device which is in hexagonal grid. So lot of research is going on to simulate the hexagonal representation. Three of those methods will be explained in this section. By using this we will be able to mimic the hexagonal grid by utilizing the rectangular grid itself.

4.1 Mimic Hexagonal Pixels Using Square Pixels

In this, He [16] proposed a method by delaying the alternate TV lines by half a pixel width. In this method pixels will be in square shape and is simple in the case of implementation also (Fig 3 (a)). But the equidistant property of the hexagonal grid is not maintained in this. From the figure it can be seen that the horizontal and vertical distance is coming to 1 unit whereas the diagonal will be $\sqrt{3}/2$.
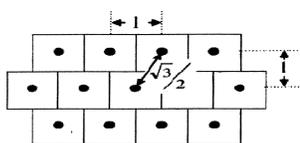
Fig 3 (a) Hexagonal structure using half pixel shift   Fig 3 (b) Rectangular pixels on a hexagonal sampling grid
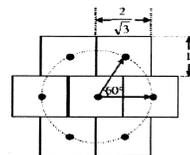
Staunton[13] proposed a new approach in which the central pixel which is the sampling point has all its neighboring pixels arranged in a circular shape as in Fig 3 (b). In this approach all the sampling point pixel are equally distant from each other. Distance between the horizontal points is $2/\sqrt{3}$ and the angle between two neighboring points will be 60°. The aspect ratio of equally sized rows and columns will be $2/\sqrt{3}:1$ in this approach.

4.2. Pseudo Hexagonal Pixel

Another approach was from Yuan [18] who evaluated the visual effect of hexagonal and square pixel. He simulated both lattices in which hexagonal pixel is called as hyperpel and was simulated using the square grid itself. But the results were very bad due to the vagueness in the screen resolution.

Yabushita [19] took this idea and extended to create a pseudo hexagonal structure which was also made from square pixels in the aspect ratio of 12:14. Later Middleton and Sivaswamy [20,21] done a great work which was a new approach in mimicking the hexagonal grid. In their work they introduced the resampling of square image into a hexagonal image.

4.3 Mimic Hexagonal Structure

In this, one hexagonal pixel means four square pixels and the equivalent grey level value is the average of these pixels [22]. This mimicking scheme will preserve the consistent neighborhood property. But by using the averaging method for calculating the grey value, resolution become lesser. Another important property known as equidistant property is also not met in this approach.

4.4 Virtual Hexagonal Structure

Wu [23] created a breakthrough in mimicking the hexagonal structure. In this he is using a virtual spiral architecture in which a spiral architecture is used during the processing part. The normal image in the traditional square grid is mapped into virtual spiral architecture and does the processing. Once the processing is done it is converted back into square grid and is displayed. It is different from all the above mimicking methods because it will neither create any distortion nor reduce resolution.

5   HEXAGONAL STRUCTURE ADDRESSING

Till we have discussed about representing and mimicking the hexagonal pixels to simulate the hexagonal grid. But we can't use the traditional row and column approach for labeling hexagonal pixels. We need to have proper coordinate system for efficiently addressing and storing hexagonal data. Research works in this area will be explained in the following section.

5.1   2-Axes coordinate addressing scheme

Luczak and Rosenfield [24], proposed the two axes oblique coordinate system (Fig 4) to address hexagonal structure. Here they use two basis vectors which are orthogonal to each other for representing the hexagonal pixel as in the figure. Any point in the two dimensional space can be represented using this system. Each point is represented as a unique ordered pair of the vectors and can easily converted to and fro from Cartesian coordinate system.
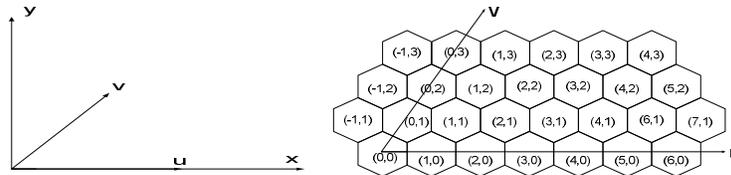
Fig 4 2-axes coordinate system for hexagonal structure

5.2. Three-Coordinate Symmetrical Coordinate Frame

In the previous method they used two vectors for addressing the data. Her[25] proposed a three coordinate system which is denoted as $*R^3$, for representing the hexagonal data as in Fig 5 (a). In this, the distance between the neighboring pixels is one unit.
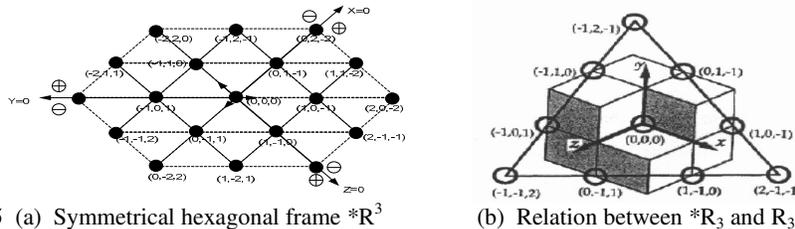
Fig 5 (a) Symmetrical hexagonal frame $*R^3$ (b) Relation between $*R_3$ and R_3

This method is very much connected with the 3-dimensional Cartesian frame R^3. The figure (Fig 5 (b)) gives a better understanding of the frame. All geometrical properties and the theoretical studies done can be easily transferred to $*R^3$ from R^3. Her [25] has done many works based on this frame especially in deriving the affine transformation. Geometrical operations in this will be simple due to its relationship with the 3-dimensional Cartesian frame R^3. The symmetry property of the hexagonal grid is also preserved in this coordinate system.

5.2 Single Indexing System

Another addressing scheme was introduced by Middleton and Sivaswamy [6] which is called as Spiral Architecture (Fig 6 (a)) based on single dimensional addressing system. In this the addressing starts spirally from the middle of the image in powers of seven. Spiral architecture consists of this addressing scheme and two operations defined on it which are spiral addition and spiral multiplication which corresponds to translation of the image and rotation of the image respectively.
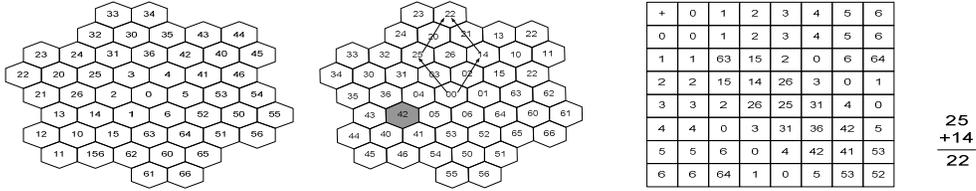
Fig 6 (a) Spiral addressing   Fig.5.5. (a) Hexagonal image structure with indices (b) Balanced ternary addition

Generalized Balanced Ternary System is modified by Middleton and Sivaswamy [6] and proposed the single index system for addressing pixels for hexagonal image processing as in Fig 6(b) and 6 (c). For finding the neighboring pixels spiral addition operation[6] is used. The spiral address of the whole image can be found by following the direction of the spiral rotation as in the figure Fig 7.
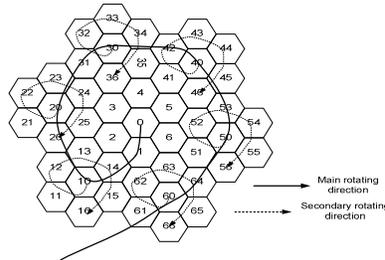
Fig 7 Spiral rotation direction

For representing an image in spiral addressing scheme we require 7^{α} hexagons, where 'α' denotes the level of spiral architecture. If 'α' is 1 it is in the first level of spiral architecture and it contains 7 elements. There is a relation to know approximately how many levels are required to represent an image in spiral architecture.

$$\alpha = \frac{\log M + \log N}{\log 7} \tag{1}$$

Where 'α' is the number of levels
M is the number of rows in the image
N is the number of columns in the image

The addressing scheme using spiral addressing scheme have many advantages over other addressing schemes as well as from square image processing. Storage space required in this is very less and geometric transformations are very easy because of its origin at the centre. It also gives consistent connectivity and unique neighborhood which is different from the traditional connectivity and neighborhood relationship.

6   CONCLUDING REMARKS

From the above discussions and explanations it as clear that there will be improvement while processing with hexagonal sampling. Since there is no dedicated hardware available for hexagonal based image capturing and display, conversion has to be done from square to hexagonal image before hexagonal image processing. Spiral addressing scheme gives a very good approach for trying the hexagonal image processing. The result will be clear only when we have the hardware for acquisition and display which works in hexagonal grid. A few companies are trying to build such devices, let's hope that it will come in the near future.

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